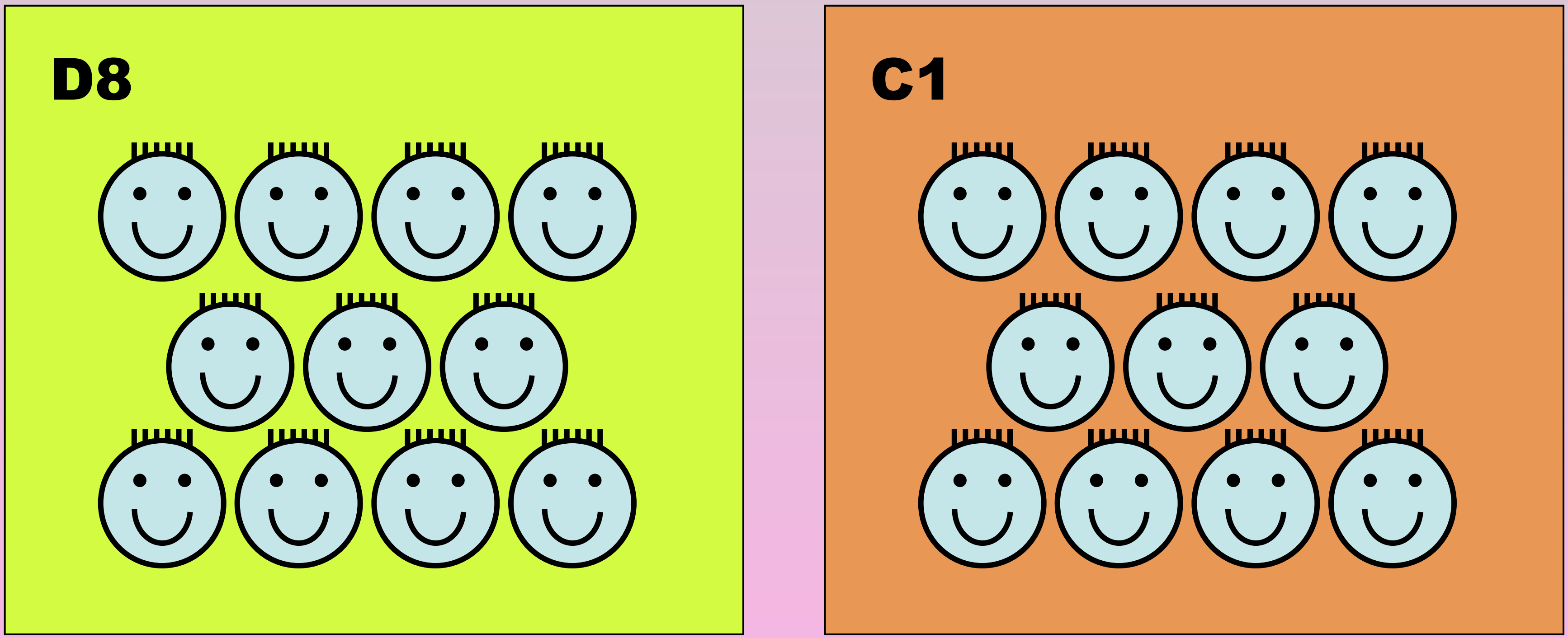
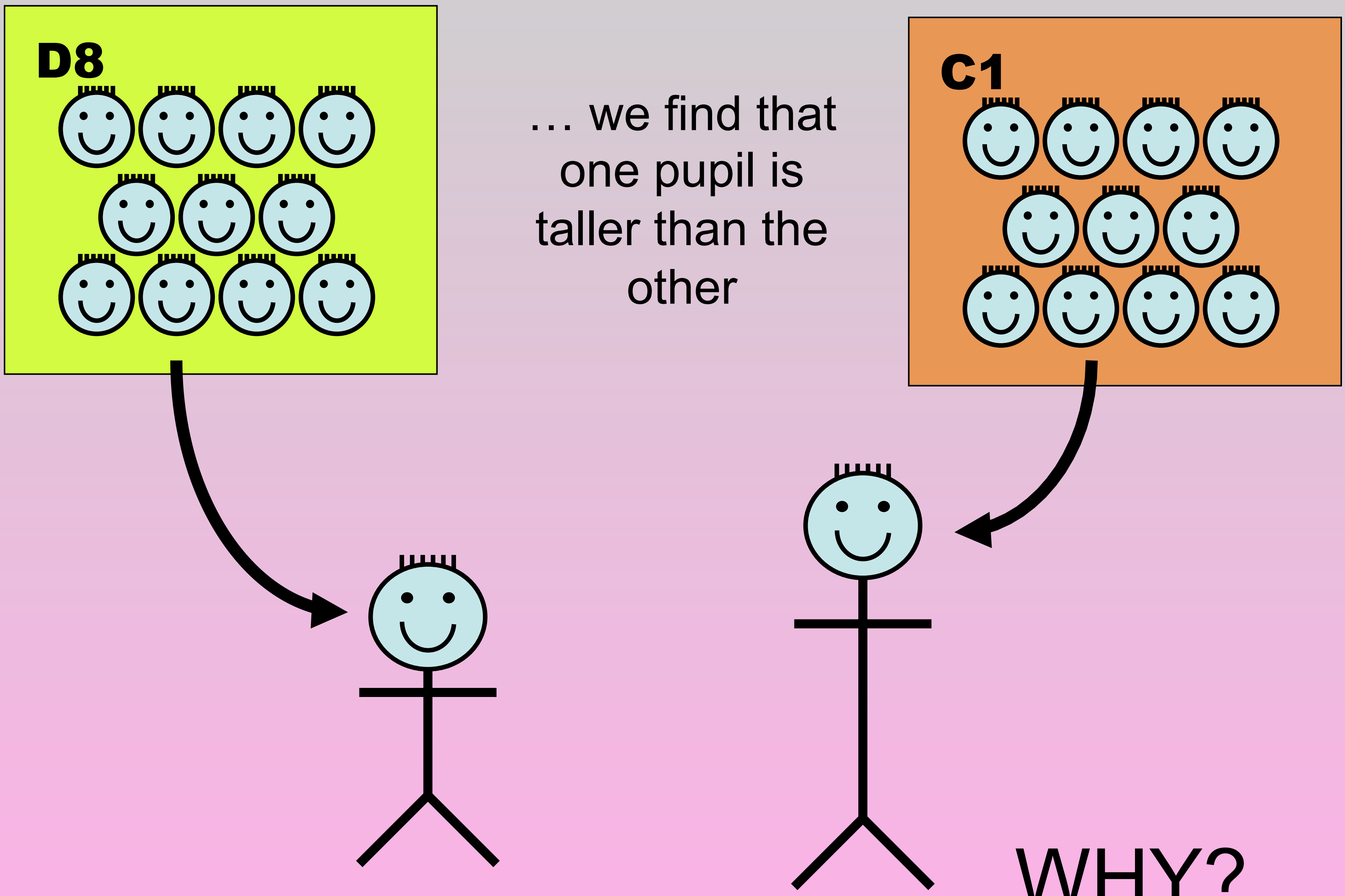
t-Test

Are our results reliable enough to support a conclusion?

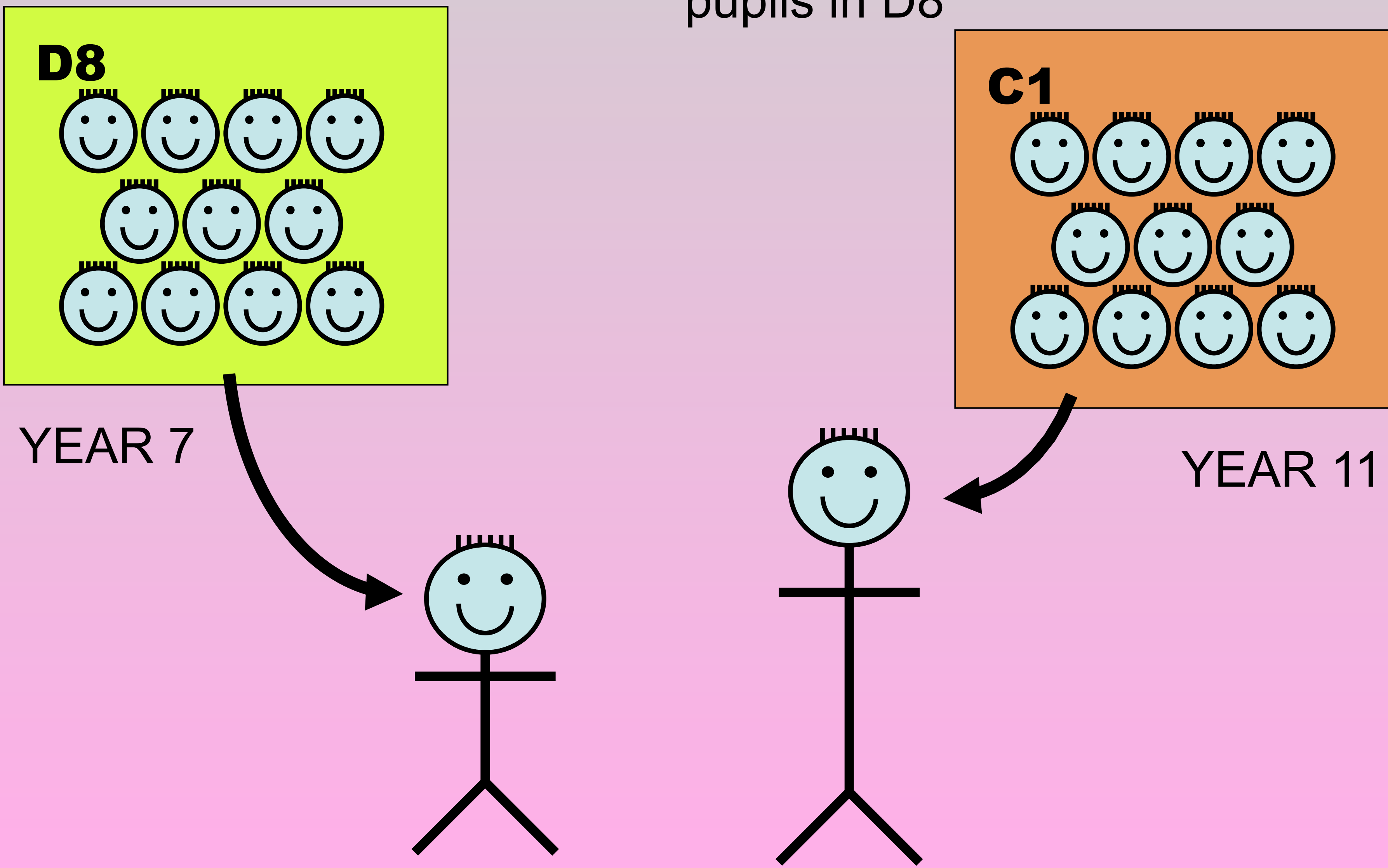
Imagine we chose two children at random from two class rooms…



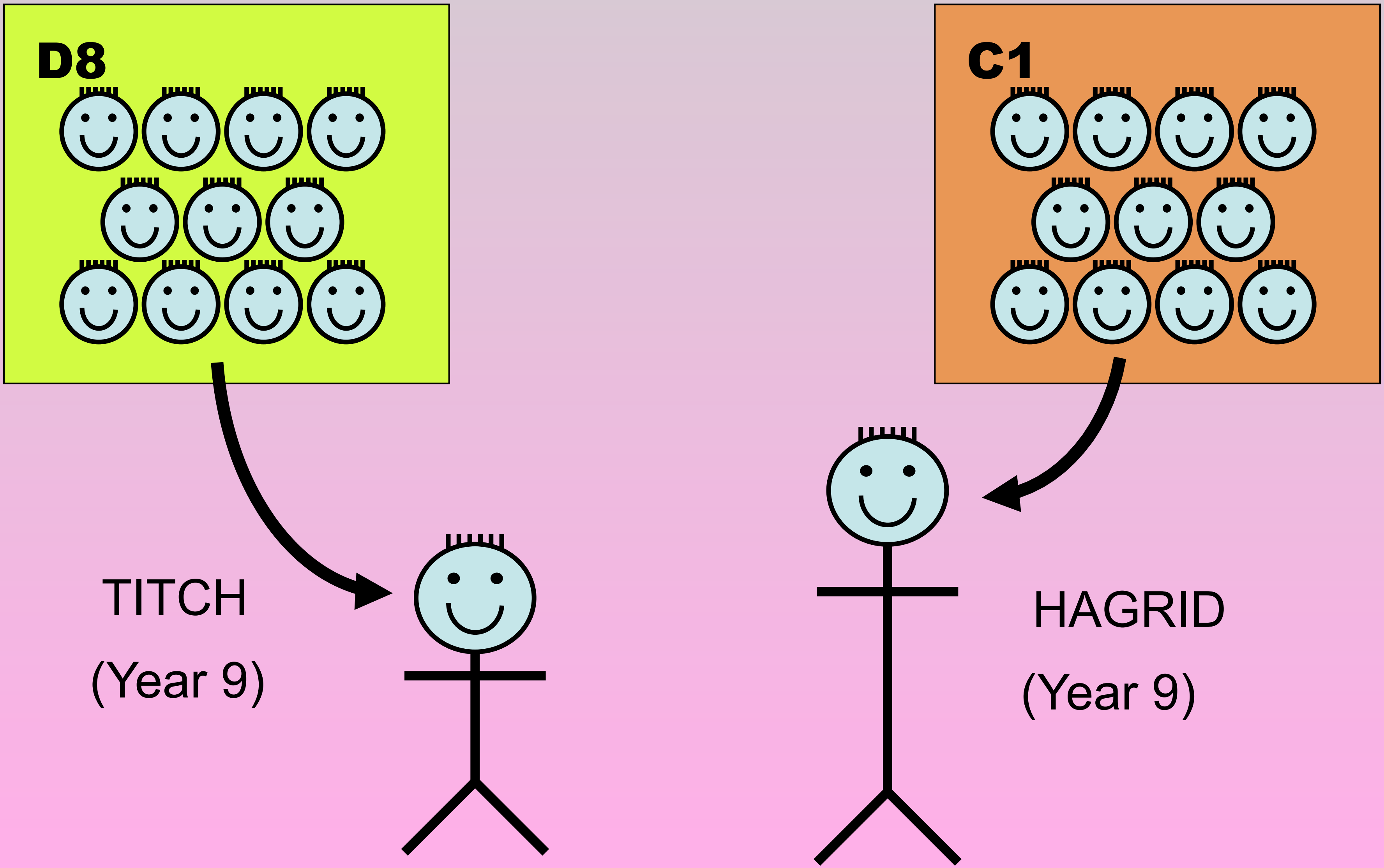
… and compare their height …


D8
C1
… we find that one pupil is taller than the other
WHY?

REASON 1: There is a significant difference between the two groups, so pupils in C1 are taller than pupils in D8



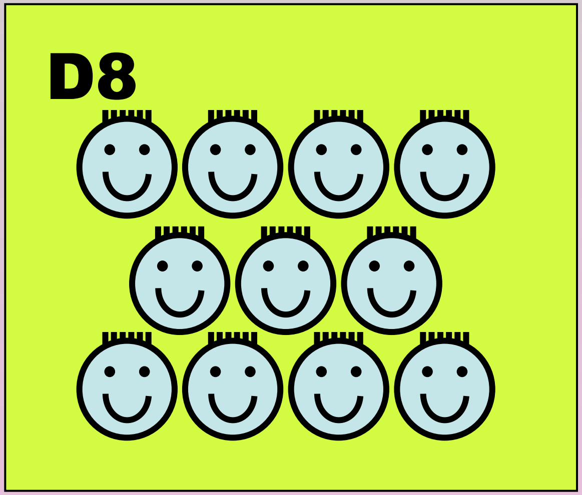
REASON 2: By chance, we picked a short pupil from D8 and a tall one from C1



How do we decide which reason is most likely?

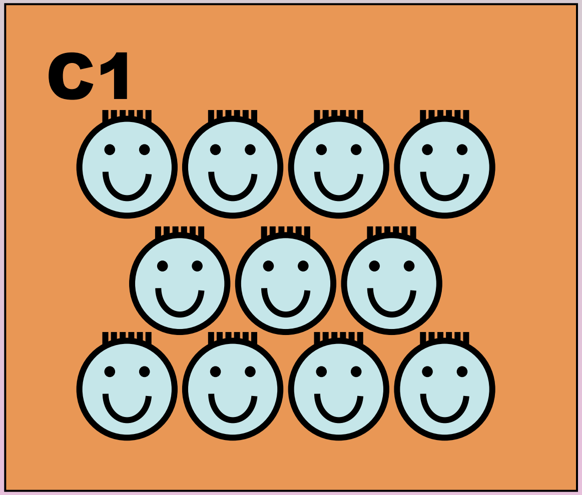
MEASURE MORE STUDENTS!!!

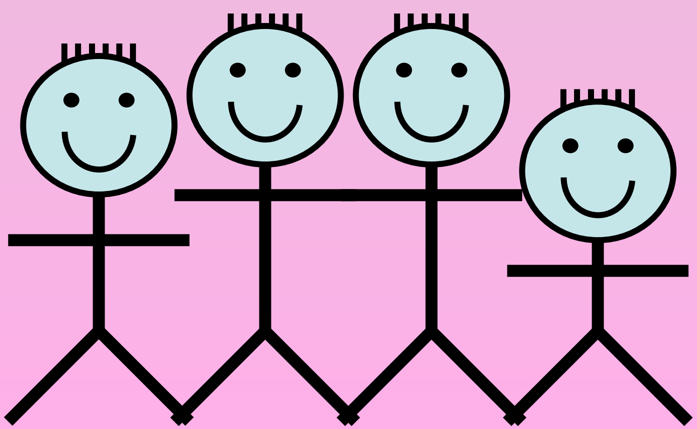
If there is a significant difference between the two groups…

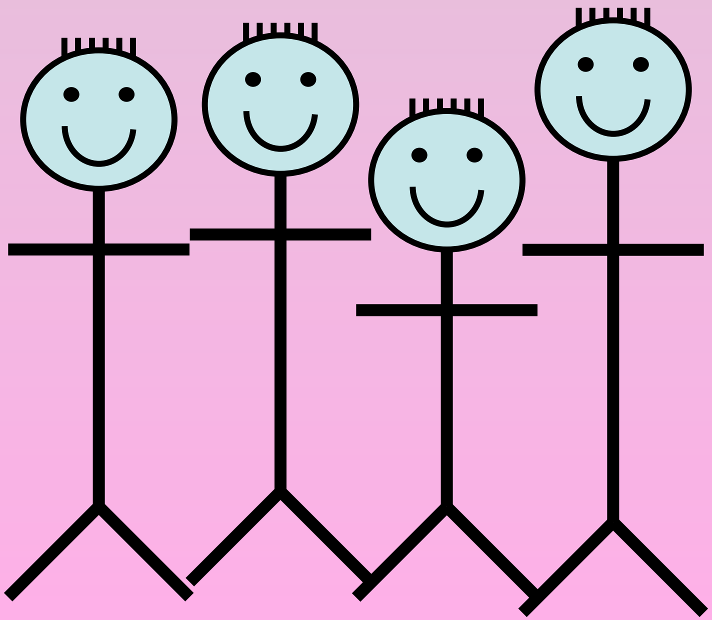


… the average or <u>mean</u> height of the two groups should be very…

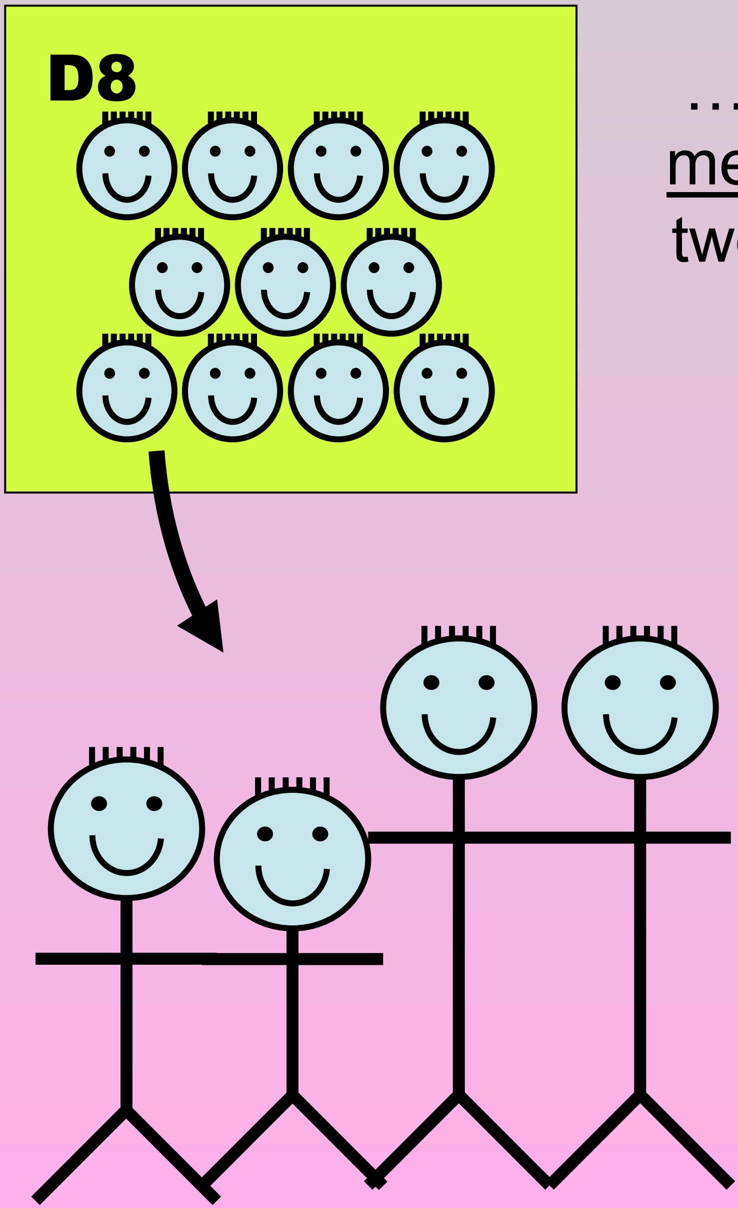
… DIFFERENT





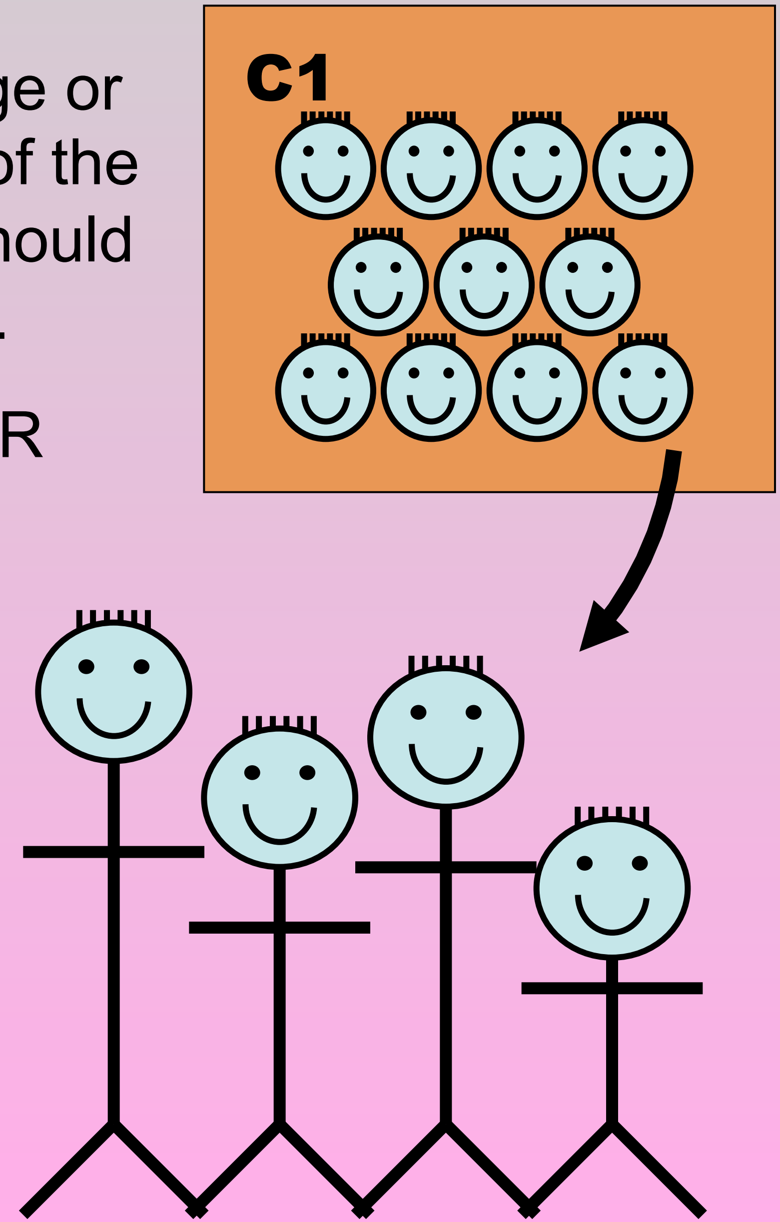


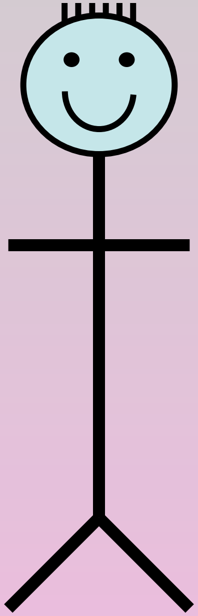
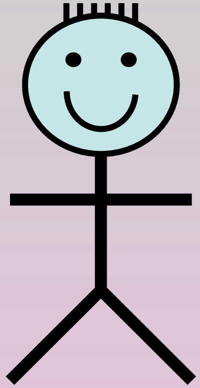
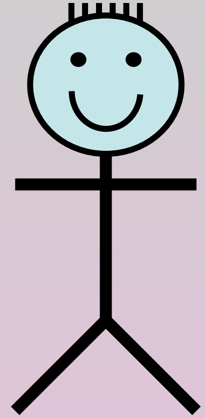
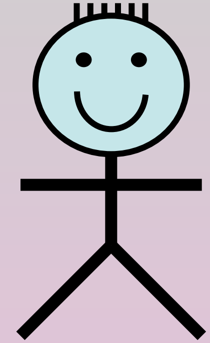
If there is no significant difference between the two groups…



… the average or <u>mean</u> height of the two groups should be very…

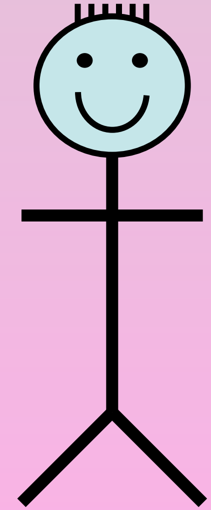
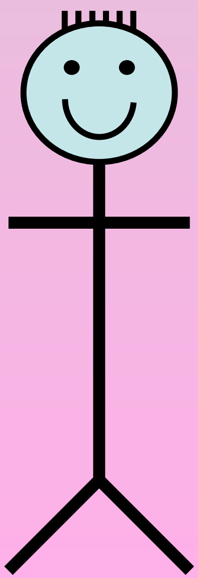
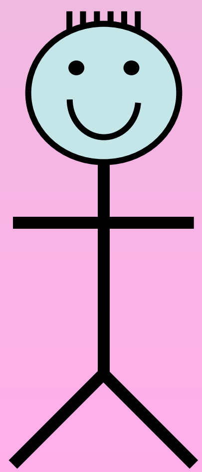
… SIMILAR



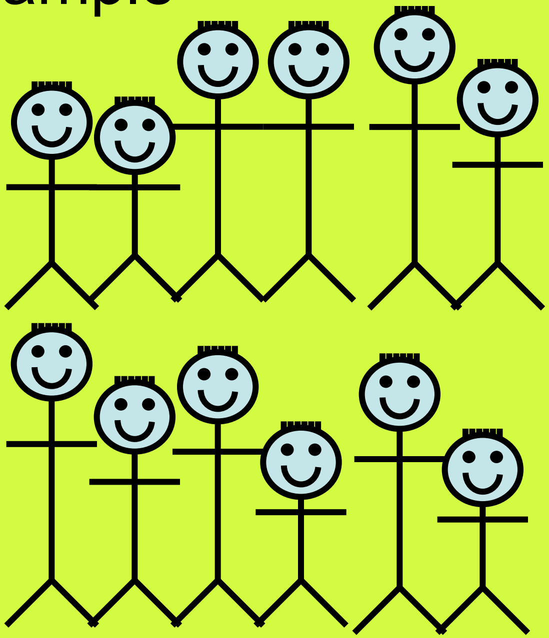
Remember:

Living things normally show a lot of variation, so…

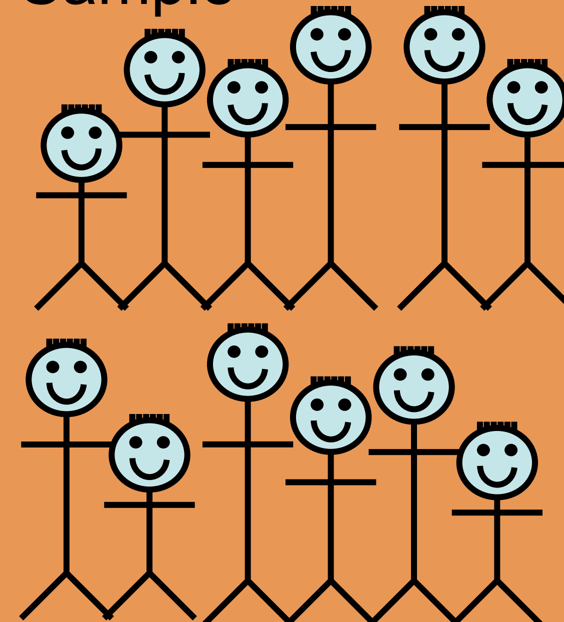
It is *VERY* unlikely that the mean height of our two samples will be exactly the same

C1 Sample



Average height = 162 cm

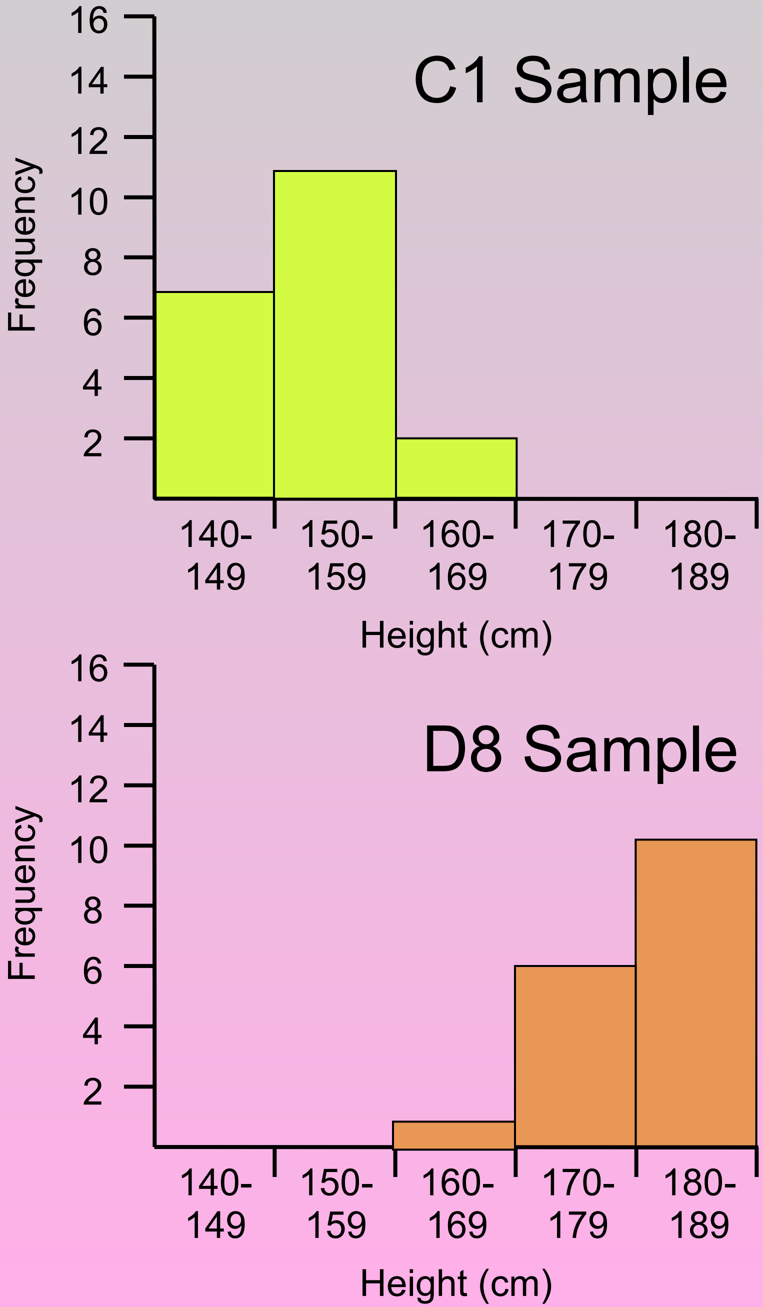
D8 Sample



Average height = 168 cm

Is the difference in average height of the samples large enough to be significant?

We can analyse the spread of the heights of the students in the samples by drawing *histograms*



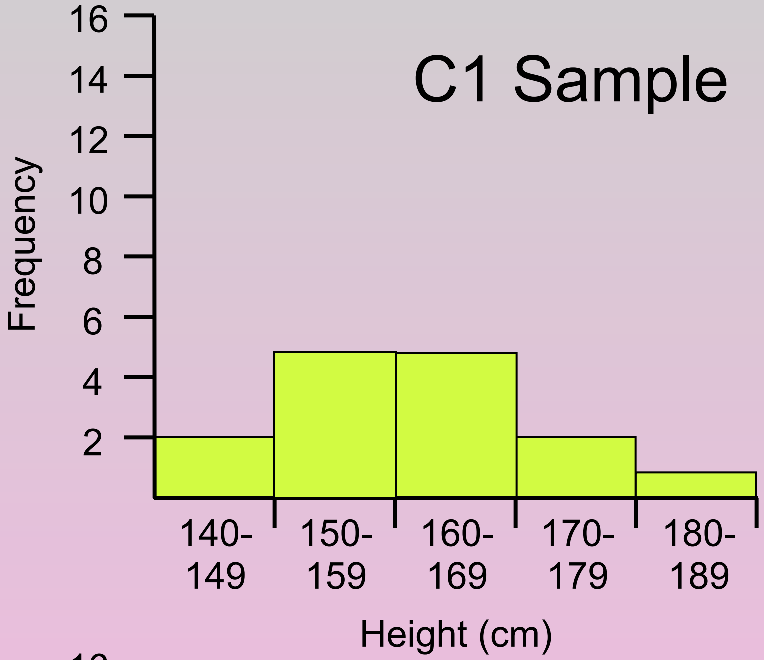
Here, the ranges of the two samples have a small overlap, so…

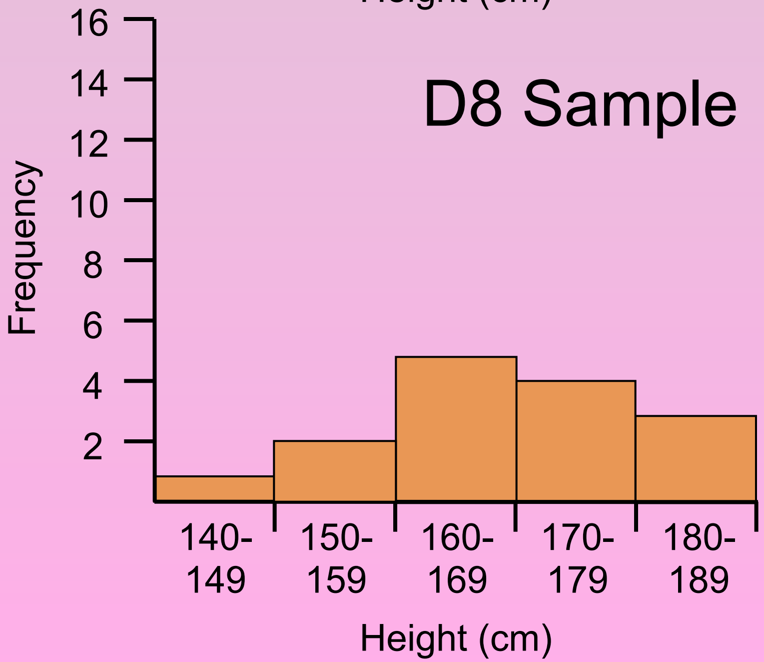
… the difference between the means of the two samples *IS* probably significant.

Here, the ranges of the two samples have a large overlap, so…

… the difference between the two samples may *NOT be* significant.

The difference in means is possibly due to *random sampling error*





To decide if there is a significant difference between two samples we must compare the *mean height* for each sample…

… and the *spread* of heights in each sample.

Statisticians calculate the *standard deviation* of a sample as a measure of the spread of a sample

You *can* calculate standard deviation using the formula:

$$S_x = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n - 1}}$$

Where:

S**x** is the standard deviation of sample

Σ stands for 'sum of'

x stands for the individual measurements in the sample

n is the number of individuals in the sample

Student's *t*-test

The Student's *t*-test compares the averages and standard deviations of two samples to see if there is a significant difference between them.

We start by calculating a number, *t*

t can be calculated using the equation:

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}}$$

Where:

$\bar{x}_1$ is the mean of sample 1
s_1 is the standard deviation of sample 1
n_1 is the number of individuals in sample 1
$\bar{x}_2$ is the mean of sample 2
s_2 is the standard deviation of sample 2
n_2 is the number of individuals in sample 2

Worked Example: Random samples were taken of pupils in C1 and D8

Their recorded heights are shown below…

	Students in C1					Students in D8				
Student Height (cm)	145	149	152	153	154	148	153	157	161	162
	154	158	160	166	166	162	163	167	172	172
	166	167	175	177	182	175	177	183	185	187

Step 1: Work out the mean height for each sample

C1: $\bar{x}_1 = 161.60$ D8: $\bar{x}_2 = 168.27$

Step 2: Work out the difference in means

$$\bar{x}_2 - \bar{x}_1 = 168.27 - 161.60 = 6.67$$

Step 3: Work out the standard deviation for each sample

C1: $s_1 =$ 10.86

D8: $s_2 =$ 11.74

Step 4: Calculate s^2/n for each sample

C1: $$\frac{(s_1)^2}{n_1} = 10.86^2 \div 15 = 7.86$$

D8: $$\frac{(s_2)^2}{n_2} = 11.74^2 \div 15 = 9.19$$

Step 5: Calculate $\sqrt{\dfrac{(s_1)^2}{n_1} + \dfrac{(s_2)^2}{n_2}}$

$$\sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}} = \sqrt{(7.86 + 9.19)} = 4.13$$

Step 6: Calculate *t* (Step 2 divided by Step 5)

$$t = \frac{\bar{x}_2 - \bar{x}_1}{\sqrt{\dfrac{(s_1)^2}{n_1} + \dfrac{(s_2)^2}{n_2}}} = \frac{6.67}{4.13} = 1.62$$

Step 7: Work out the number of degrees of freedom.
Sample size of both groups minus 2

$$d.f. = n_1 + n_2 - 2 = 15 + 15 - 2 = \quad 28$$

NOTE:

On a t-value Table the (p) probability tells us that chance alone could make a difference. If p=0.5, the difference is due to chance 50% of the time. This is not a significant difference in statistics.

However, if p=0.05, the probability that the difference is due to chance is only 5% (risk or alpha level). There is a 95% chance the differences is due to one set being different from another. There is a significant difference

Step 8: Find the critical value of *t* for the relevant number of degrees of freedom on a t-Test table

Use the 95% (p=0.05) confidence limit

Alpha or risk level is = 2.05

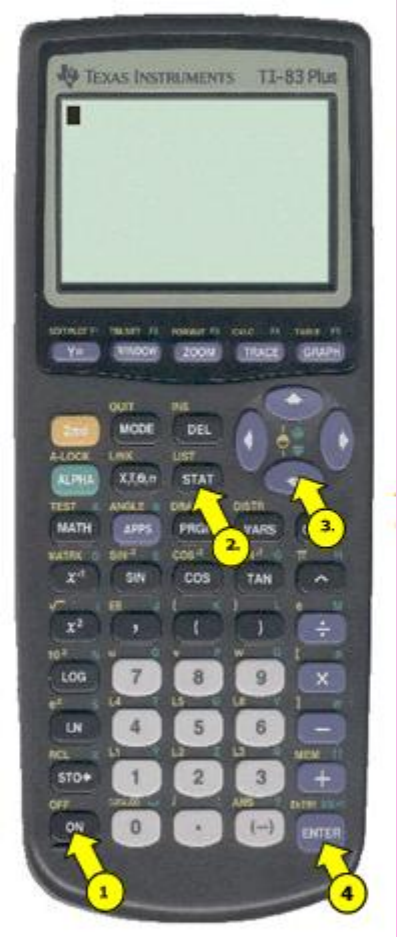
NOTE: Watch the table for 1-tail or 2-tail. You want 2-tail

df	.10	.05	.025
1	3.078	6.314	12.706
2	1.886	2.920	4.303
3	1.638	2.353	3.182
4	1.533	2.132	2.776
5	1.476	2.015	2.571
6	1.440	1.943	2.447
7	1.415	1.895	2.365
8	1.397	1.860	2.306
9	1.383	1.833	2.262
10	1.372	1.812	2.228
11	1.363	1.796	2.201
12	1.356	1.782	2.179
13	1.350	1.771	2.160
14	1.345	1.761	2.145
15	1.341	1.753	2.131
16	1.337	1.746	2.120
17	1.333	1.740	2.110
18	1.330	1.734	2.101
19	1.328	1.729	2.093
20	1.325	1.725	2.086
21	1.323	1.721	2.080
22	1.321	1.717	2.074
23	1.319	1.714	2.069
24	1.318	1.711	2.064
25	1.316	1.708	2.060
26	1.315	1.706	2.056
27	1.314	1.703	2.052
28	1.313	1.701	2.048
29	1.311	1.699	2.045

Our calculated value of t *of 1.62 is below the critical value for 28d.f. (2.05), therefore,*

This leads us to conclude that we can accept the null hypothesis and state that there is **NOT** a statistically significant difference between *the height of students in samples from C1 and D8*

- The **null hypotheses** typically corresponds to a general or default position.
- For example, the null hypothesis might be that there is no relationship between two measured phenomena

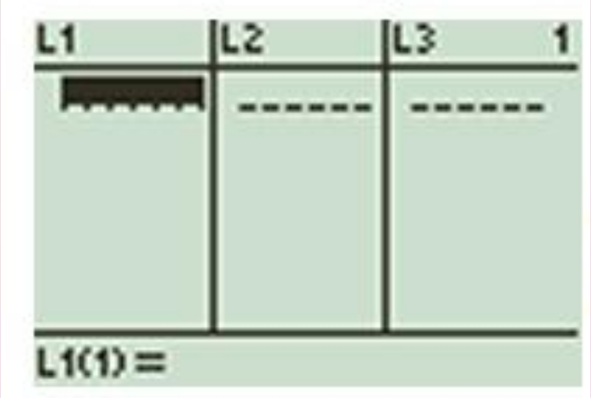


Using the Calculator
t-Test

t-Test

- t-Test for two samples, unpaired data, two-tailed test, population mean unknown

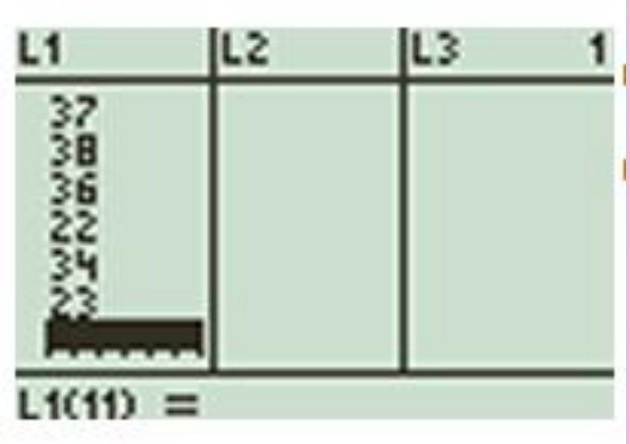
t-Test



- Press “On”
- Press “STAT”
  - L1 is going to be the list of the first sample data
  - L2 will be the list of the second sample of data
  - Don’t need a third list but if necessar it is there

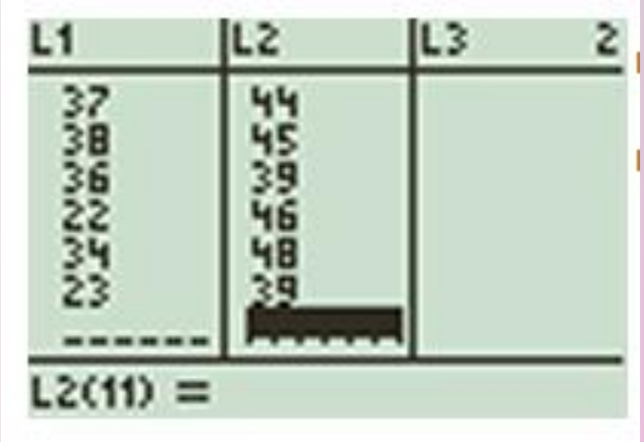
t-Test

- Add the data using the calculator keypad after every data point
- Press “Enter”
- Once you have all the data in sample 1 move to L2



t-Test

- Your second set will look like this



- If you make a mistake entering data, highlight the data point and overwrite the entry
- If you want to clear the whole list move the cursor to the L1 or L2 and use the “CLEAR” button

t-Test

- Press “STAT” and you will see this screen

- Use the arrow key on the keypad move the cursor to the right from “EDIT” to “TESTS

t-Test

- Scroll down to the number 4 using the keypad to select “4:2-SampTTest

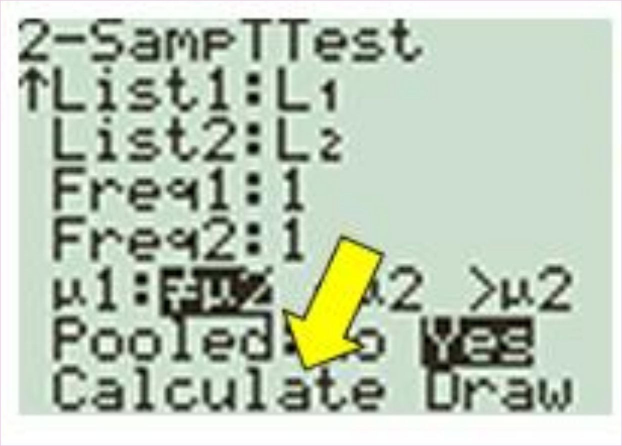
- Press “Enter”

t-Test

- You now have a choice to make
  - DATA will be selected and flashing. Leave it
  - Make sure “u1” is selected as shown (this is a Two tailed selection) and Pooled: Yes

t-Test

- Scroll down a little more using the arrows on the keyboard and select “Calculate”
- Press ”ENTER”



t-Test

- The calculation is complete.

T = is the value we have been calculated